Leg Forces on a Simple Staked Chair

Ars Mechanica
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DISCLAIMER

The design equations below are for a simplified chair and do not take into account all possible combinations of force and geometry. Use at your own risk and without any warranty of accuracy or correctness. Consult a licensed professional engineer if your design is critical.

Force Analysis

We make the following assumptions in the design:

1. The contribution due to floor friction is negligible.
2. The seat is sufficiently thick as to be rigid.
3. The chair leg is uniform in cross-section.
4. Each leg supports an equal amount of the bearing weight.

The impact of these assumptions will be discussed as we progress through the force analysis. We start by constructing the free-body diagram (FBD) of an idealized leg with no stretchers shown in Figure 1.

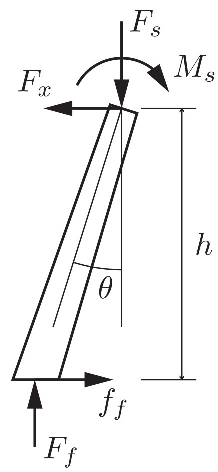

Figure 1: Free-body diagram of an idealized leg with no stretcher.

We treat the leg as a beam that is fixed rigidly into the seat. From the forces in Figure 1 we arrive at the following system of equilibrium equations

$$F_x: \quad f_f - F_x = 0 \tag{1}$$
$$F_y: \quad F_f - F_s = 0 \tag{2}$$
$$M_0: \quad f_f h - F_f \left(\frac{h}{\cos\theta}\right) \sin\theta - M_s = 0 \tag{3}$$

We make the assumption that the weight supported by the chair is distributed equally amongst all the legs, which is likely not the case, but is a reasonable initial assumption. Therefore the seat force $F_s = W/N$, where N is the number of legs and W is the weight of the sitter. We make the further assumption that the force due to friction is negligible $f_f = 0$. The effect of this assumption is to increase the moment at the seat M_s, which leads to a more conservative estimate of stress in the leg.

We first calculate the moment at the point where the leg enters into the seat mortise.

$$M_s = F_f \left(\frac{h}{\cos\theta}\right) \sin\theta \tag{4}$$

The negative cancels as we see that the direction of the moment in Figure 1 is opposite of reality. From Equation (2) and the seat force assumption described above we see that

$$F_f = F_s = \frac{W}{N} \tag{5}$$

This gives us the moment in the chair leg as

$$M_s = \frac{W}{N}\left(\frac{h}{\cos\theta}\right)\sin\theta \tag{6}$$

The axial force along the leg F_a is arrived at by decomposing F_f into components normal and parallel to the angle of the chair leg

$$F_a = F_f \cos\theta = \frac{W}{N}\cos\theta \tag{7}$$

At this point, we have the forces acting on the leg and can proceed to perform the stress analysis of the leg to determine failure loads.

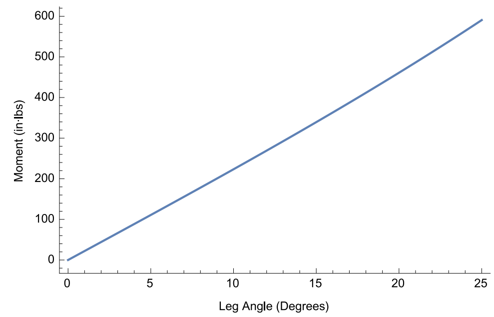

Figure 2: Moment acting at leg mortise for $N = 3$, $h = 19$, and $F_s = 200$ lb.

Stress analysis

The stress in beam due to bending is given by

$$\sigma_b = \frac{Mc}{I}$$

where M is the bending moment, c is the distance from the neutral axis to the outermost fiber, and I is the moment of inertia or the 2^{nd} moment of area. We have M from equation 6 above, all that is left is to calculate the moment of inertia I and solve for the stress.

Since the mortise is circular where the leg enters into the seat, we will assume a circular moment of inertia which is given by

$$I = \frac{\pi}{2}r^4 \tag{8}$$

Combining with the equation above, we arrive at an expression for the maximum tensile bending stress

$$\sigma_b = \frac{2Wh}{N\pi r^3}\tan\theta \tag{9}$$

The axial force is always compressive, which serves to lower the tensile bending stress and is added by superposition giving

$$\sigma_{tensile} = \frac{W}{N}\left(\frac{2h}{\pi r^3}\tan\theta - \frac{1}{\pi r^2}\cos\theta\right) \tag{10}$$

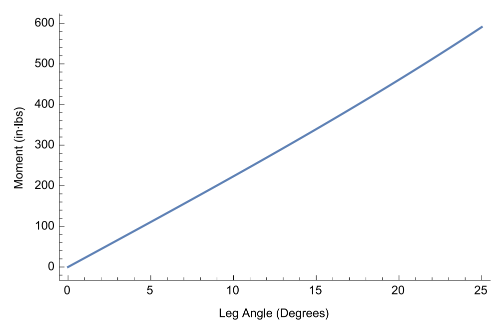

Figure 3: Stress at leg mortise for $N = 3$, $h = 19$, $F_s = 200$ lb, and $r = 1.125$ in.

Table 1: Properties of selected hardwoods.

Species	Modulus of Rupture (psi)
White Oak	18,400
Red Oak	18,100
Ash	15,000
Hard Maple	15,800
Hickory	20,200

As we can see in Figure 3, the stress in the leg for a three legged stool supporting 200 pounds that has a 19 in tall leg, with a 21 degree angle is approximately 1677 psi. This is significantly below the failure stress for some common woods that we might want to use for legs. Some example material properties are given in Table 1. The strength listed is the modulus of rupture, which is a reasonable measure of wood failure stress under a nominally bending loading. We can see for the static case considered here that the oaks have a factor of safety of nearly 10.

To understand the effect of leg length, the stress at the mortise for a $r = 1.125$ in leg mortise, a $\theta = 17°$ leg angle, and a seat height varying from 18 to 27 inches is given in Figure 4.

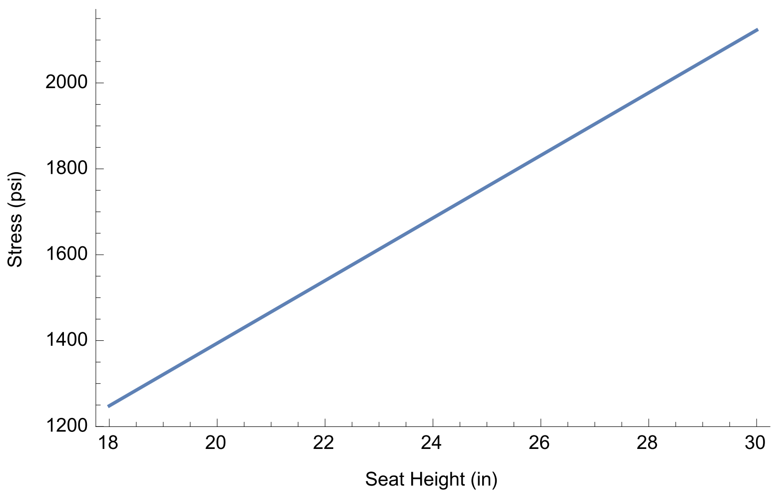

Figure 4: Stress at leg mortise for $N = 3$, $\theta = 17°$, $F_s = 200$ lb, and $r = 1.125$ in.

Even for a seat height of 27 inches, which corresponds to a leg length of 28.25 inches, the stress at the mortise is only 2000 psi. This gives a factor of safety of nearly 9 for the oaks and 10 for the hickories. In short, nearly any leg is capable of withstanding a reasonable static load. Even doubling the load for a dynamic event, increases the stress by slightly more than twice the static load, giving factors of safety in the 5 to 6 for oaks and 8 to 9 for the hickories.